## **1.1 Exponential Functions**

## Problems Worksheet



- 1. A particular animal has a population of 2200 at the start of 2013. The population is growing continuously according to the rule  $P = Ae^{0.054t}$ , where P is the number of animals t years after the start of 2013.
  - a. What is the population at the start of 2016?
  - b. How fast is the population growing after 4.5 years?
  - c. At the start of 2018, the population begins to decrease continuously by 7.15% every year. When will the population be less than 500 if this trend continues?

- 2. The decay of an element is given by  $M = Ae^{kt}$ , where M is in grams and t is in days. The half-life (time taken for mass to halve) is known to be 98.6 days.
  - a. Calculate the rate of decay.
  - b. How long will it take for 200g of the element to first decay below 10g?
  - c. After the time found in part b, what is the rate of change of the amount of the element if starting with 200g?

- 3. A radioactive material decays according to  $P = P_0 e^{-0.1234t}$ , where  $P_0 = 0.4\mu g$ , and t is measured in hours.
  - a. What is the half-life of the material?
  - b. After how long will it take for there to be 5% left of the original amount of the material?
- 4. Atmospheric pressure (in millibars) is given by  $P = 1013.25e^{kd}$ , where *d* is distance (m) above sea level. a. What is *k* if atmospheric pressure is 898.746 millibar at 1000 metres?

b. A mountain is 3155m high. What is the atmospheric pressure at the top of this mountain?

c. Kathmandu has an atmospheric pressure of 850 millibar. How far is this city above sea level?

d. If climbing at an altitude of 2000m, what is the rate of change of atmospheric pressure at this height?

- 5. For the following functions, state the equation of any asymptotes and determine the gradient at x=2
  a. f(x) = e<sup>x</sup>
  - b.  $f(x) = e^{4x+2} + 3$
  - c.  $f(x) = -e^{-8x+3} 3$
  - d.  $f(x) = -e^{-3x^2 + 3x} 7$

6. The graph of  $f(x) = ae^{b(x+c)} + d$  is shown below.



- a. Determine a, b, c and d given that |a| = |b| = 1
- b. If the graph g(x) is a tangent line which intercepts f(x) at (-2,2), what is the equation of the line?

7. Prove the following using first principles.

$$\frac{d}{dx}(e^x) = e^x$$

- 8. Assume the height of a rocket, in kilometres, roughly follows the equation  $h = -e^{-0.04t+5} + e^5$  at t seconds for  $t \ge 0$ .
  - a. If the equation was written in the form  $Ae^{kx} + c$ , what would be the values of A, k and c?
  - b. Will the rocket ever reach a height of  $e^5$  kilometres? Explain why using mathematical reasoning.
  - c. What is the rate of increase of height at t=50
  - d. At what height will the rate be 1 km per second?
  - e. At what time does the rocket increase its height at the greatest rate?